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Commutators in the Middle Nucleus

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Abstract— In this paper, we assume that R is an antiflexible ring with commutators in the middle nucleus. First we prove that, let B =∑ (N, N) + (N, N) R, B is an ideal of R such that BA = 0 where A is an associator ideal of R. If R is prime then R is associative or (N, N) = 0. Using these results, we prove that, (i) a semiprime antiflexible ring is commutative, (ii) a prime antiflexible ring is either associative or commutative and finally (iii) a semi-prime antiflexible ring is isomorphic to a direct sum of a semi prime associative ring and a semiprime commutative ring.

Index Terms— Associator, commutator, simple, nucleus, antiflexible rings.

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1 Introduction

A. Thedy in 1971 [5] studied rings R which satisfy the identities ((R, R), R, R) = 0, ((x, y, x), y) = 0, for all x, y in R and either (R, (R, R), R) = 0 or (R, R, (R, R)) = 0. His main result is that prime rings satisfying these identities must be either commutative or associative

In this paper, we assume antiflexible ring with commutators in the middle nucleus and prove a semiprime antiflexible ring is commutative, a prime antiflexible ring is either associative or commutative and a semiprime antiflexible ring is isomorphic to a direct sum of a semi prime associative ring and a semiprime commutative ring.

2 Preliminaries

The ring R is said to be antiflexible if it satisfies the identity (x,y,z) - (z,y,x)=0.

A ring R can be defined as an antiflexible ring if it satisfies the identity

A(x, y, z) = (x, y, z) – (z, y, x) = 0 …(1)

Thoughout the remainder of this section, we assume that R is an antiflexible ring of charactetristic ≠ 2, and

(x, x, x) = 0 …(2)

is an identity in R.

With the aid of (1), we obtain the identity as a linearization of (2) as

B(x, y, z) = (x, y, z) + (y, z, x) + (z, x, y) = 0 ...(3)

The following identities Teichmuller and semi-Jacobi which holds in any ring

C (w, x, y, z) = (w x, y, z) – (w, x y, z) + (w, x, y z) – w (x,y,z)–(w,x,y)z=0 … (4)

D(x, y, z)=(x y, z)–x(y, z)–(x, z) y–(x, y, z)–(z, x, y) + (x, z, y)= 0 …(5)

The identity (5) can also be written as

(x y, z) = x (y, z) + (x, z) y + B (x, y, z) – (x, z, y) –(y, z, x) …(6)

The nucleus

N = {n∈ R / (R, R, n) = (n, R, R) = (R, n, R) = 0}

And the centreC ={c ∈ R / (c, R) = 0}.

Def 2.1:

A ring R is said to be simple if whenever A is an ideal of R, then either A=R or A = 0.

Def 2.2:

The associator (x, y, z) is defined by

(x, y, z) = (xy) z – x (yz), for all x, y, z in a ring.

Def 2.3:

The commutator (x, y) is defined by

(x, y) = x y – y x, for all x,y in a ring.

Suppose n, n′ ∈ N, x, y, z ∈ R.

Then n n′ (x, y, z) = n (x n′, y, z)
= (n. x n′, y, z)
= (nx. n′, y, z)
= n′ (nx, y, z)
= n′n(x, y, z)
[using,(nx, y,z)=n (x,y,z)=(x n,y,z), n∈N]

Thus (n, n′) (x, y, z) = 0.

So that (N, N) (R, R, R) = 0. …(7)

The associator ideal B of R is defined as

B = Σ (R, R, R) + (R, R, R) R.

Let B(x, y, z) = (x, y, z) + (y, z, x) + (z, x, y).

In every ring we have the identity

((x,y),z)+((y,z),x)+((z,x),y)=B(x,y,z)-B(x,z,y) …(8)

Now we first prove some properties of the nucleus in R.

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3 Main Section:

Lemma 3.1:

Let B = Σ (N, N) + (N, N) R. Then B is an ideal of R such that BA=0

Proof :

We substitute y = n, z = n′ in (8), then we get

((x, n), n′) + (n, n′), x) + ((n′, x), n) = B (x, n, n′) – B (x, n′, n) = 0.

Since (N, R) ⊂ (R, R) ⊂ N,

we have ((n, n′), x) ⊂ Σ (N, N).

Thus ((N, N), R) ⊂ Σ (N, N). … (9)

Now we take n ∈ N, w, x, y, z ∈ R.

Then (5) implies (n w, x) = n (w, x) + (n, x) w + (n, w, x) + (x, n, w) – (n, x, w).

Now (n w, x) ∈ N because of (6) and n (w, x) ∈ N as a result of

(n x, y, z) = n (x, y, z) = (x n, y, z), n ∈ N …(10)

Moreover, three associators on R.H.S are zero. Now we insert the equation as arbitrary associator form. Then we get

((n, x) w, y, z) = 0.

Hence (10) implies (n, x) (w, y, z) = 0. …(11)

Now substitute n = (n′, n′′) in (11).

Thus ((n′, n′′), x) (R, R, R) = 0.

That is, ((N, N), R) (R, R, R) = 0. …(12)

We observe that (N, N) ⊂ N,

So that B is clearly right ideal of R.

Using (9), it shows that R (N, N) ⊂ B.

But then (8) and (11) suffice to prove R.

(N, N) R ⊂ B.

Consequently B is an ideal of R. ♦

Theorem 3.2:

If R is prime then R is associative or (N, N) = 0.

Proof:

(N, N) A = 0 follows readily from 7.

Then (N, N) R. A = (N, N). R A ⊂ (N, N) A = 0.

Thus B A = 0.

Since R is prime, either B = 0 or A = 0.

If A = 0, then R is associative.

If B = 0, then (N, N) = 0. ♦

Theorem 3.3:

R is isomorphic to a subdirect sum of a semi prime associative ring and a semiprime antiflexible ring and (N, N) = 0 holds.

Proof :

It follows from Theorem 3.2 that

$(B \cap A)\,(B \cap A) \subset BA = 0$.

Thus semiprime implies $B \cap A = 0$.

If B^* is a maximal ideal which contains B and has zero intersection with A and if A^* is a maximal ideal which contains A but has zero intersection with B^*, then $B^* \cap A^* = 0$. As in [8, proof of Theorem 4, "Hentzel. I. R and Smith .H.F., Semiprime locally (-1,1) rings with minimal condition", Algebras, Graphs and Geom., 2(1985), 26-52]

We can write that R/B^* and R/A^* are semiprime.

This completes the Proof of the theorem. ♦

Theorem 3.4 :

(i) A semiprime antiflexible ring is commutative.

(ii) A prime antiflexible ring is either associative or commutative, and

(iii) A semiprime antiflexible ring is isomorphic to a subdirect sum of a semiprime associative ring and a semiprime commutative ring.

Proof : The following identity holds in all rings

(a, b) a = (a, b, a) + (a, b a). … (13)

By commuting (13) with b, we get

((a, b) a, b) = ((a, b, a), b) + ((a, b a), b) = 0.

But ((a, b) a, b) = (a, b) (a, b).

In a semiprime ring the only center element which squares to zero is zero.

Thus (a, b) = 0.

so that R is commutative.

The second assertion follows from Theorem (3.2) and the first assertion.

The last assertion follows from Theorem (3.3). ♦

4. Conclusion:

From this we conclude that, an antiflexible ring R is that if it is semiprime then is commutative, if it is a prime then is either associative or commutative.

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